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# On infinite direct sums of lifting modules

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## Abstract

© 2017 World Scientific Publishing Company. The aim of the present article is to investigate the structure of rings R satisfying the condition: for any family {Si|i ϵ N} of simple right R-modules, every essential extension of ⊗i ϵ NE(Si) is a direct sum of lifting modules, where E(-) denotes the injective hull. We show that every essential extension of ⊗i ϵ NE(Si) is a direct sum of lifting modules if and only if R is right Noetherian and E(S) is hollow. Assume that M is an injective right R-module with essential socle. We also prove that if every essential extension of M(ϵ N) is a direct sum of lifting modules, then M is ∑-injective. As a consequence of this observation, we show that R is a right V-ring and every essential extension of S(ϵ N) is a direct sum of lifting modules for all simple modules S if and only if R is a right ∑-V-ring.

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## Keywords

H-ring, Hollow module, Lifting module, Noetherian ring, QF-ring, ∑ -injective module, ∑ -lifting module, ∑ -V-ring

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